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# Anisotropic Cosmological Models with Variable G and Decaying Vacuum Energy

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**Abstract** We present Bianchi type-I cosmological models with a perfect fluid source and time-dependent gravitational and cosmological constants based on new exact solutions of Einstein's equations. The perfect fluid is chosen to obey a barotropic equation of state. The models obtained represent a radiation dominated phase and a dust era. In some of the models the expansion changes from a decelerating phase to an accelerating one and these models asymptotically tend to the de Sitter universe.

**Keywords** Bianchi type universe · Vacuum energy · Variable G and Λ

## 1 Introduction

In the past few years, evidence has mounted that some constants, which were earlier treated as true constants, are no longer constants in cosmology. In Einstein's theory of gravity Newtonian gravitational 'constant' G and cosmological 'constant' Λ are considered as fundamental constants. The gravitational 'constant' G plays the role of coupling constant between geometry of space and matter in Einstein's field equations. In an evolving universe, it appears natural to look at this constant as a function of time. Dirac [12] and Dicke [11] have suggested a time-varying gravitational constant. The Large Number Hypothesis (LNH) proposed by Dirac [13, 14] leads to a cosmology where G varies with cosmic time. There have been many extensions of Einstein's theory of gravitation, with time dependent G, in order

The paper is dedicated to late Prof. S.R. Roy, Ex-Head, Department of Mathematics, Banaras Hindu University, Varanasi, India.

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to achieve a possible unification of gravitation and elementary particle physics. Canuto and Narlikar [9] have shown that the G varying cosmology is consistent with whatever cosmological observations are presently available. Levit [19], Beesham [5] and Abdel-Rahman [1] have discussed the possibility of an increasing G. Massa [22]) has proposed a model in favour of increasing G depending on a maximal power hypothesis (MPH). Now, it has been proposed to link the variation of gravitational constant with that of variable cosmological term-$\Lambda$ within the framework of general relativity keeping the form of field equations formally unchanged. Cosmological models with variable-G and $\Lambda$ have been studied by a number of researchers [2, 6, 7, 17, 18, 20, 35] for homogeneous and isotropic Friedmann–Robertson–Walker (FRW) line-element.

It has been a subject of considerable interest of cosmologists to study the universe in the early stages of its evolution. Spatially homogeneous and anisotropic cosmological models which provide a richer structure, both geometrically and physically, than the FRW models play significant role in the description of early universe. The theoretical arguments and experimental data, which support the existence of an anisotropic phase that approaches an isotropic one [10], lead to consider the models with anisotropic background. The simplest anisotropic models are of Bianchi type-I which are homogeneous also. In such models there exist flat spatial sections but direction-dependent rates of expansion or contraction. To study the possible effects of anisotropy of the early universe on present day observations, many researchers have investigated the Bianchi type-I model from different points of view. Bianchi type-I cosmological models for various matter distributions have been discussed by Saha [28–33] in a series of papers. Bianchi type-I models with variable G and $\Lambda$ have been studied by Beesham [6], Vishwakarma et al. [36], Vishwakarma [37], Maharaj and Naidoo [21], Arbab [3, 4], Pradhan and Yadav [23].

In this paper, we investigate the evolution of an anisotropic Bianchi type-I universe with variable G and $\Lambda$ in the presence of a perfect fluid obeying barotropic equation of state. Universe models representing different phases (radiation dominated and matter dominated) of evolution have been presented. Some of the models are found to be accelerating at later epochs and they asymptotically tend to de Sitter universe.

## 2 The Field Equations and Assumptions

We consider Bianchi type-I space-times in orthogonal form represented by

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2. \tag{1}$$

The space-times are filled with perfect fluid given by the energy-momentum tensor

$$T_{ij} = (\rho + p)v_i v_j + p g_{ij}, \tag{2}$$

$\rho$, $p$ and $v_i$ being respectively the energy density, isotropic pressure and the unit flow vector of the fluid distribution. $v_i$ is given by $v_i = -\delta_{i4}$. We assume that the matter content of the universe satisfies the perfect gas equation of state

$$p = \omega\rho, \quad 0 \leq \omega \leq 1. \tag{3}$$

The Einstein field equations for the time-dependent G and $\Lambda$ can be written as

$$R_{ij} - \frac{1}{2}R g_{ij} = -8\pi \mathrm{G}(t) T_{ij} + \Lambda(t) g_{ij}. \tag{4}$$

Covariant divergence of (4) leads to

$$8\pi \mathrm{G}\left[\rho_4+(\rho+p)\left\{\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right\}\right]+8\pi\rho \mathrm{G}_4+\Lambda_4=0. \tag{5}$$

Here, and throughout, the suffix "4" stands for ordinary time-derivative of the concerned quantity. It is easy to observe from (5) that presence of matter is essential for a time-varying $\Lambda$. In fact, the existence of time-varying cosmological term $\Lambda(t)$ in the field equations (4) accounts that apart from the usual energy-momentum tensor of the matter content $T_{ij}$ of the universe, there is an additional term $-\frac{\Lambda}{8\pi \mathrm{G}}g_{ij}$ which may be regarded as the energy-momentum tensor of vacuum with its energy density $\rho_v$ and homogeneous, isotropic pressure $p_v$ satisfying the equation of state $p_v=-\rho_v=-\frac{\Lambda}{8\pi \mathrm{G}}$.

The usual conservation law for energy-momentum tensor $T^j_{i;j}=0$ leads to

$$\rho_4+(\rho+p)\left\{\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right\}=0. \tag{6}$$

Equation (5) together with (6) puts G and $\Lambda$ in some kind of coupled fields given by

$$8\pi\rho \mathrm{G}_4+\Lambda_4=0. \tag{7}$$

The field equations (4) for the metric (1) with matter distribution (2) yield

$$8\pi \mathrm{G}p-\Lambda=-\frac{B_{44}}{B}-\frac{C_{44}}{C}-\frac{B_4C_4}{BC}, \tag{8}$$

$$8\pi \mathrm{G}p-\Lambda=-\frac{A_{44}}{A}-\frac{C_{44}}{C}-\frac{A_4C_4}{AC}, \tag{9}$$

$$8\pi \mathrm{G}p-\Lambda=-\frac{A_{44}}{A}-\frac{B_{44}}{B}-\frac{A_4B_4}{AB}, \tag{10}$$

$$8\pi \mathrm{G}\rho+\Lambda=\frac{A_4B_4}{AB}+\frac{B_4C_4}{BC}+\frac{A_4C_4}{AC}. \tag{11}$$

Equation (6) gives on integration

$$\rho=\frac{k'}{(ABC)^{\omega+1}} \tag{12}$$

where $k'>0$ is a constant of integration. From (8, 9) and (10), we get only two independent equations:

$$\frac{A_4}{A}-\frac{B_4}{B}=\frac{k_1}{ABC}, \tag{13}$$

$$\frac{B_4}{B}-\frac{C_4}{C}=\frac{k_2}{ABC}, \tag{14}$$

where $k_1\neq 0$ and $k_2\neq 0$ are constants of integration. We define a function $f(t)$ as $f=ABC$, and hence our average scale factor is $f^{1/3}$. Equations (13) and (14) then yield

$$3\frac{A_4}{A}=\frac{f_4}{f}+\frac{(2k_1+k_2)}{f}, \tag{15}$$

$$3\frac{B_4}{B} = \frac{f_4}{f} + \frac{(k_2 - k_1)}{f}, \tag{16}$$

$$3\frac{C_4}{C} = \frac{f_4}{f} - \frac{(k_1 + 2k_2)}{f}. \tag{17}$$

We introduce volume expansion $\theta$ and shear $\sigma$ as usual

$$\theta = v^i{}_{;i}, \qquad \sigma^2 = \frac{1}{2}(\sigma_{ij}\sigma^{ij}), \quad \sigma_{ij} \text{ being shear tensor.}$$

In the above the semicolon stands for covariant differentiation.

For the Bianchi type-I metric expressions for the dynamical scalar come out to be [29]

$$\theta = \frac{f_4}{f}, \qquad \sigma = \frac{k}{\sqrt{3}f}, \qquad k^2 = k_1^2 + k_2^2 + k_1 k_2. \tag{18}$$

In analogy with a FRW universe, we define a generalized Hubble parameter $H$ and the generalized deceleration parameter $q$ as

$$3H = \frac{f_4}{f}, \tag{19}$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -\frac{H_4}{H^2} - 1. \tag{20}$$

Equations (8–11) can be written in terms of $H, \sigma$ and $q$ as

$$8\pi \mathrm{G} p - \Lambda = H^2(2q - 1) - \sigma^2, \tag{21}$$

$$8\pi \mathrm{G}\rho + \Lambda = 3H^2 - \sigma^2. \tag{22}$$

Equation (22) implies that

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} - \frac{8\pi \mathrm{G}\rho}{\theta^2} - \frac{\Lambda}{\theta^2}.$$

Therefore, $0 \leq \frac{\sigma^2}{\theta^2} \leq \frac{1}{3}$ and $0 \leq \frac{8\pi \mathrm{G}\rho}{\theta^2} \leq \frac{1}{3}$ for $\Lambda \geq 0$.

Thus the presence of a positive $\Lambda$ puts the restriction on the upper limit of anisotropy whereas a negative $\Lambda$ will contribute to the anisotropy. From (21), we obtain

$$\frac{d\theta}{dt} = -12\pi \mathrm{G} p - \frac{\theta^2}{2} - \frac{3\sigma^2}{2} + \frac{3\Lambda}{2}.$$

Thus a positive $\Lambda$ will arrest the rate of decrease and a negative $\Lambda$ would promote it. Also, the rate of decrease in an anisotropic background is smaller in comparison to that in the isotropic one. We have $\sigma_4 = -3\sigma H$ implying that $\sigma$ decreases in an evolving universe. Equations (3) and (7–11) constitute six equations in seven unknowns $A$, $B$, $C$, $\rho$, $p$, $\Lambda$ and G. To have determinate solution(s), we require one more condition, which we take as

$$\mathrm{G} = \mathrm{G}_0 + \alpha f^{\omega - 1}, \tag{23}$$

where $\mathrm{G}_0$ ($\geq 0$) and $\alpha$ are constants. This form of G was arrived at by Vishwakarma [37] by considering $\Lambda \propto \sigma^2$. The choice (23) allows diverse physical situations for consideration.

For positive $\alpha$, G happens to be decreasing with the expansion of the universe, while for a negative $\alpha$, G turns out to be slowly increasing in an expanding universe which may slow down the late time acceleration and G may even be negative. These are valid for all possible values of $\omega$ except $\omega = 1$ ($\rho = p$) in which case G is trivially a constant like that when $\alpha = 0$. Of course, the rate of above increase/decrease is different for different values of $\omega$. In particular, the rate of decrease is faster but increase rate is slower in matter dominated universe ($\omega = 0$) than the corresponding ones in the radiation universe ($\omega = 1/3$). Putting (23) in (7), we obtain

$$\Lambda = \Lambda_0 + \frac{4\pi\alpha k'(\omega - 1)}{f^2} \tag{24}$$

so that the cosmological term $\Lambda$ tends asymptotically to the constant of integration $\Lambda_0$ which may be determined with the help of (22). Further, depending upon the sign of $\Lambda_0$ or otherwise the cosmological term $\Lambda$ might be negative as well in which case possibility of an oscillatory mode of evolution may also exist [28, 30].

Using (3, 12) and (18–22) we obtain

$$\left(\frac{f_4}{f}\right)_4 + \frac{k^2}{f^2} = \frac{-12\pi \mathrm{G}(\omega + 1)k'}{f^{\omega+1}}. \tag{25}$$

Equation (25) together with (23) reduces to

$$\left(\frac{f_4}{f}\right)_4 + \frac{k^2}{f^2} = -\frac{12\pi k'\mathrm{G}_0(\omega + 1)}{f^{\omega+1}} - \frac{12\pi k'\alpha(\omega + 1)}{f^2}, \tag{26}$$

which, on integration, gives

$$t + t_0 = \int [k^2 + 12\pi k'\alpha(\omega + 1) + 24\pi k'\mathrm{G}_0 f^{1-\omega} + Nf^2]^{-1/2} df. \tag{27}$$

Here $N$ and $t_0$ are constants of integration. Right hand side of (27) is difficult to integrate, in general. However, it can be integrated in different phases of evolution. When $\omega = 0$ (i.e. $p = 0$), (27) can be integrated to give the following:

$$Nf = 2a + b\cosh\sqrt{N}T \quad \text{for } N > 0 \quad \text{and} \quad 144\pi^2 k'^2\mathrm{G}_0^2 > N(k^2 + 12\pi\alpha k'), \tag{28}$$

$$Nf = 2a + b'\sinh\sqrt{N}T \quad \text{for } N > 0 \quad \text{and} \quad 144\pi^2 k'^2\mathrm{G}_0^2 < N(k^2 + 12\pi\alpha k'), \tag{29}$$

$$Nf = Ne^{\sqrt{N}T} + 2a \quad \text{for } N > 0 \quad \text{and} \quad 144\pi^2 k'^2\mathrm{G}_0^2 = N(k^2 + 12\pi\alpha k'), \tag{30}$$

$$(-N)f = b\sin\sqrt{-N}T - 2a \quad \text{for } N < 0 \quad \text{and} \quad 144\pi^2 k'^2\mathrm{G}_0^2 > N(k^2 + 12\pi\alpha k') \tag{31}$$

and

$$f = -aT^2 + \frac{c}{4a} \quad \text{for } N = 0. \tag{32}$$

Here

$$a = -6\pi k'\mathrm{G}_0,$$
$$b = \left\{144\pi^2 k'^2\mathrm{G}_0^2 - N(k^2 + 12\pi\alpha k')\right\}^{1/2},$$
$$b' = \left\{-144\pi^2 k'^2\mathrm{G}_0^2 + N(k^2 + 12\pi\alpha k')\right\}^{1/2},$$

$$c = k^2 + 12\pi\alpha k',$$
$$T = t + t_0.$$

When $\omega = 1/3$, (27) gives on integration for $k^2 + 16\pi\alpha k' = 0$:

$$f^{2/3} = \sqrt{\frac{24\pi k' G_0}{N}} \sinh\left(\frac{2\sqrt{N}T}{3}\right) \quad \text{for } N > 0, \tag{33}$$

$$f^{2/3} = \sqrt{\frac{24\pi k' G_0}{-N}} \sin\left(\frac{2\sqrt{-N}T}{3}\right) \quad \text{for } N < 0. \tag{34}$$

For $N = 0$ and $k^2 + 12\pi\alpha k'(\omega + 1) = 0$, (27) reduces to

$$f = \left\{\left(\frac{1+\omega}{2}\right)\sqrt{24\pi k' G_0} T\right\}^{2/(\omega+1)},$$

which is the model considered by Vishwakarma [37].

## 3 Discussions

We have derived a number of model universes (28–34) in Sect. 2. Most of these, as we will see below, tend to a de Sitter universe at late times. The models (28–32) have pressureless fluid distribution characterizing the epoch dominated by dust matter. As already pointed out, we have used (22) for finding $\Lambda$, and as such $\Lambda_0$ has been determined precisely in each model.

The model (28) is described by

$$\rho = \frac{Nk'}{2a + b\cosh\sqrt{N}T},$$
$$\theta = \frac{\sqrt{N} b \sinh\sqrt{N}T}{2a + b\cosh\sqrt{N}T},$$
$$\sigma = \frac{Nk}{\sqrt{3}(2a + b\cosh\sqrt{N}T)},$$
$$G = G_0 + \frac{N\alpha}{2a + b\cosh\sqrt{N}T},$$
$$\Lambda = \frac{N}{3} - \frac{4\pi\alpha k' N^2}{(2a + b\cosh\sqrt{N}T)^2},$$
$$q = 2 - 3\left[\frac{\cosh\sqrt{N}T(2a + b\cosh\sqrt{N}T)}{b\sinh^2\sqrt{N}T}\right].$$

The model has singularity at

$$T = \frac{1}{\sqrt{N}}\cosh^{-1}\left(-\frac{2a}{b}\right) \equiv T_0 \text{ (say)}.$$

We note that when $\alpha > 0$, G is always positive and $\Lambda < 0$ for

$$T_0 < T < \frac{1}{\sqrt{N}}\cosh^{-1}\left(\frac{\sqrt{12\pi\alpha k'N}-2a}{b}\right).$$

In case $\alpha < 0$, we have $\Lambda > 0$ always. In this case, G becomes negative during

$$T_0 < T < \frac{1}{\sqrt{N}}\cosh^{-1}\left(-\frac{2a}{b}-\frac{\alpha N}{b\mathrm{G}_0}\right).$$

Thus in the beginning of the model, either $\Lambda$ is negative or G is negative. The model starts with a big-bang from its singular state with $\rho$, $|\mathrm{G}|$, $|\Lambda|$ and $\sigma$ all infinite and evolves to isotropy with $\mathrm{G} \to \mathrm{G}_0$ and $\theta \to \sqrt{N}$ as $T \to \infty$. At $T = T_0$, deceleration parameter $q = 2$, and in the limit $T \to \infty$, $q = -1$ which is very encouraging because the current observations of SNeIa and CMB favour accelerating models ($q < 0$) [37]. The cosmological term $\Lambda(t)$ tends asymptotically to $N/3$ so that at late times, the model tends to a de Sitter universe with $H = \sqrt{\Lambda/3}$. The vacuum energy density comes out to be

$$\rho_v = \frac{N(2a+b\cosh\sqrt{N}T)^2 - 12\pi\alpha k'N^2}{24\pi(2a+b\cosh\sqrt{N}T)\{\mathrm{G}_0(2a+b\cosh\sqrt{N}T)+\alpha N\}}.$$

At $T = T_0$, $|\rho_v|$ is infinite and $\rho_v \to \frac{N}{24\pi\mathrm{G}_0}$ as $T \to \infty$. At the beginning of the model vacuum energy density is half of the matter density whereas at late times it dominates the matter density. The time $T_\Lambda$ after which the cosmological term $\Lambda$ starts dominating corresponds to $\Lambda = 8\pi\mathrm{G}\rho$ [8]. For the model we obtain

$$T_\Lambda = \frac{1}{\sqrt{N}}\cosh^{-1}\left[\frac{-4a \pm \sqrt{4a^2+36\pi\alpha k'N}}{b}\right].$$

The time $T_q$, when the expansion changes from the decelerating phase to an accelerating one, is found to be

$$T_q = \frac{1}{\sqrt{N}}\cosh^{-1}\left[\frac{-3a \pm \sqrt{9a^2-2b^2}}{b}\right].$$

Behaviours of the model (29) are similar to those of (28). For the model (30), cosmological parameters are given by

$$\rho = \frac{k'}{e^{\sqrt{N}T}+\frac{2a}{N}},$$

$$\theta = \frac{\sqrt{N}e^{\sqrt{N}T}}{e^{\sqrt{N}T}+\frac{2a}{N}},$$

$$\sigma = \frac{k}{\sqrt{3}(e^{\sqrt{N}T}+\frac{2a}{N})},$$

$$\mathrm{G} = \mathrm{G}_0 + \frac{\alpha}{e^{\sqrt{N}T}+\frac{2a}{N}},$$

$$\Lambda = \frac{N}{3} - \frac{4\pi\alpha k'}{(e^{\sqrt{N}T}+\frac{2a}{N})^2},$$

$$q = -1 - \frac{6a}{Ne^{\sqrt{N}T}}.$$

The model has singularity at

$$T = \frac{1}{\sqrt{N}} \log\left(-\frac{2a}{N}\right) \equiv T_1 \text{ (say)}.$$

In this model also, when $\alpha > 0$, G is always positive whereas $\Lambda$ becomes negative during the period

$$T_1 < T < \frac{1}{\sqrt{N}} \log\left\{-\frac{2a}{N} + \sqrt{\frac{12\pi\alpha k'}{N}}\right\}.$$

For $\alpha < 0$, $\Lambda$ is always positive but G becomes negative for

$$T_1 < T < \frac{1}{\sqrt{N}} \log\left(-\frac{2a}{N} - \frac{\alpha}{G_0}\right).$$

The model starts with a big-bang with $\rho$, $|G|$, $\sigma$, $|\Lambda|$ all infinite and evolves to isotropy with $G \to G_0$ and $\theta \to \sqrt{N}$ as $T \to \infty$. At $T = T_1$, $q = 2$ and $q \to -1$ as $T \to \infty$. In this model also, $\Lambda$ tends asymptotically to $N/3$ so that we come across a de Sitter universe with $H = \sqrt{\Lambda/3}$. Vacuum energy density in this model also is infinite at the initial singularity and it tends to a constant asymptotically. The time $T_\Lambda$ for the model is given by

$$T_\Lambda = \frac{1}{\sqrt{N}} \log\left[-\frac{4a}{N} \pm \frac{1}{N}\sqrt{8a^2 - k^2N + 24\pi k'\alpha N}\right],$$

whereas the time $T_q$ is given by

$$T_q = \frac{1}{\sqrt{N}} \log\left(-\frac{6a}{N}\right).$$

We now discuss the model (31) which is described by

$$\rho = \frac{-Nk'}{b \sin\sqrt{-N}T - 2a},$$

$$\theta = \frac{b\sqrt{-N}\cos\sqrt{-N}T}{b \sin\sqrt{-N}T - 2a},$$

$$\sigma = \frac{-Nk}{\sqrt{3}(b \sin\sqrt{-N}T - 2a)},$$

$$G = G_0 - \frac{\alpha N}{b \sin\sqrt{-N}T - 2a},$$

$$\Lambda = \frac{N}{3} - \frac{4\pi\alpha k' N^2}{(b \sin\sqrt{-N}T - 2a)^2},$$

$$q = -1 - 3\left[\frac{(2a \sin\sqrt{-N}T - b)}{b \cos^2\sqrt{-N}T}\right].$$

We discuss the model in two different cases corresponding to $b > -2a$ and $b < -2a$. When $b > -2a$, the model has a singularity at

$$T = \frac{1}{\sqrt{-N}} \sin^{-1}\left(\frac{2a}{b}\right) \equiv T_2 \text{ (say)},$$

the singularity being approached as $(b \sin\sqrt{-N}T - 2a) \to O^+$. The model starts with a big bang from $T = T_2$ and the expansion decreases to become zero at $T = \pi/(2\sqrt{-N})$. Thereafter contraction starts to render $\theta$ a minimum (negative) value at

$$T = \frac{1}{\sqrt{-N}} \sin^{-1}\left(\frac{b}{2a}\right),$$

where from $\theta$ increases to become zero at $T = 3\pi/(2\sqrt{-N})$, and $\theta = -b\sqrt{-N}/(2a)$ at $T = 2\pi/\sqrt{-N}$. We observe that for $T > 2\pi/\sqrt{-N}$, $\theta$ behaves periodically with the period $2\pi/\sqrt{-N}$. When $b < -2a$, the model starts with a finite expansion at $T = 0$ and thereafter $\theta$ behaves periodically with the period $2\pi/\sqrt{-N}$. In this model, we observe that when $\alpha > 0$, $\Lambda < 0$ and $\mathrm{G} > 0$ always, whereas for $\alpha < 0$, $\Lambda$ becomes negative for

$$T_2 < T < \frac{1}{\sqrt{-N}} \sin^{-1}\left(\frac{2a - \sqrt{12\pi\alpha k' N}}{b}\right)$$

and G becomes negative during

$$T_2 < T < \frac{1}{\sqrt{-N}} \sin^{-1}\left(\frac{2a}{b} + \frac{\alpha N}{b\mathrm{G}_0}\right).$$

At $T = T_2$, each of $\rho, \theta, \sigma, |\Lambda|, |\mathrm{G}|$ is infinite and $q = 2$. At $T = 0$, $\rho, \theta, \sigma, |\Lambda|, |\mathrm{G}|$ are all finite and $q = 2$. At $T = \pi/(2\sqrt{-N})$, $\rho, \sigma, |\Lambda|$, G become finite and $q$ becomes infinite. At $T = \pi/\sqrt{-N}$, $\rho, \sigma, |\Lambda|$ and G remain finite and $q$ becomes finite ($q = 2$).

For the model (32), we have

$$\rho = \frac{1}{6\pi \mathrm{G}_0 (T^2 - \frac{c}{4a^2})},$$

$$\theta = \frac{2T}{T^2 - \frac{c}{4a^2}},$$

$$\sigma = \frac{-k}{\sqrt{3}a(T^2 - \frac{c}{4a^2})},$$

$$\mathrm{G} = \mathrm{G}_0 - \frac{\alpha}{a(T^2 - \frac{c}{4a^2})},$$

$$\Lambda = \frac{-4\pi\alpha k'}{a^2(T^2 - \frac{c}{4a^2})^2},$$

$$q = \frac{1 + \frac{3c}{4a^2T^2}}{2}.$$

We discuss the model in three cases, corresponding to $c > 0$, $c < 0$ and $c = 0$. When $c > 0$, $\alpha$ may assume both positive and negative values. For $\alpha > 0$, $\Lambda$ is always negative and

G is always positive, whereas, for $\alpha < 0$, $\Lambda$ is always positive and G becomes negative for

$$-\frac{\sqrt{c}}{2a} < T < \sqrt{\frac{c}{4a^2} + \frac{\alpha}{a\mathrm{G}_0}}.$$

In this case, a singularity exists at $T = -\sqrt{c}/(2a) \equiv T_3$ (say). The model starts with a big-bang from the singular state with $\rho, \sigma, |\Lambda|$ and $|\mathrm{G}|$ all infinite and evolves to isotropy with $\mathrm{G} \to \mathrm{G}_0$ and $\Lambda \to 0$ as $T \to \infty$. At $T = T_3$, deceleration parameter $q = 2$ and vacuum energy density is half of the matter density, whereas for $T \to \infty$, $q \sim 1/2$ and the matter density dominates the vacuum energy density.

When $c < 0$, $\alpha$ will be negative. In this case $\Lambda$ will be positive throughout and G will be negative during the period $0 < T < \sqrt{c/(4a^2) + \alpha/(a\mathrm{G}_0)}$. The model starts from the rest at $T = 0$ with $\rho, \sigma, \Lambda$ and $|\mathrm{G}|$ all finite. The rate of expansion increases to attain its maximum at $T = -\sqrt{-c}/(2a)$. After that contraction starts to become zero at $T = \infty$. At $T = 0$; $\sigma/\theta$ and $q$ are infinite and at $T = -\sqrt{-c}/(2a)$, $\sigma/\theta$ becomes finite and $q$ becomes $-1$. As $T \to \infty$, $\mathrm{G} \to \mathrm{G}_0$, $\Lambda \to 0$, $\sigma/\theta \to 0$ and $q \to 1/2$. Thus the model approaches isotropy.

Lastly, in the case $c = 0$, $\alpha$ will always be negative. $\Lambda$ is always positive and G becomes negative during $0 < T < \sqrt{\alpha/(a\mathrm{G}_0)}$. The model has a singularity at $T = 0$, where from it starts with a big-bang with $\rho, \sigma, \Lambda$ and $|\mathrm{G}|$ all infinite, and expands till $T = \infty$. As $T \to \infty$, G becomes $\mathrm{G}_0$ and $\Lambda \to 0$. The model evolves to isotropic universe asymptotically having the constant deceleration parameter $q$ (=1/2).

Model (33) is described by

$$\rho = \frac{N}{24\pi \mathrm{G}_0 \sinh^2(\frac{2\sqrt{N}}{3}T)},$$

$$\theta = \sqrt{N}\coth\left(\frac{2\sqrt{N}}{3}T\right),$$

$$\sigma = \frac{k}{\sqrt{3}}\frac{1}{(\sqrt{\frac{24\pi k'\mathrm{G}_0}{N}})^{3/2}\sinh^{3/2}(\frac{2\sqrt{N}}{3}T)},$$

$$\mathrm{G} = \mathrm{G}_0 - \frac{k^2}{16\pi k'}\frac{1}{\sqrt{\frac{24\pi k'\mathrm{G}_0}{N}}\sinh(\frac{2\sqrt{N}}{3}T)},$$

$$\Lambda = \frac{N}{3} + \frac{k^2/6}{(\frac{24\pi k'\mathrm{G}_0}{N})^{3/2}\sinh^3(\frac{2\sqrt{N}}{3}T)},$$

$$q = -1 + \frac{2}{\cosh^2(\frac{2\sqrt{N}}{3}T)}.$$

The model starts with a big-bang at $T = 0$ and expands till $T = \infty$ to have $\theta = \sqrt{N}$ ultimately. $\rho$ is infinite initially and becomes zero at the end of the model. $\Lambda$, which is initially $+\infty$, reduces to $N/3$ at the end. The model is obviously representing at late times, a de Sitter universe with $H = \sqrt{\Lambda/3}$. G is $-\infty$ at the start and it becomes $\mathrm{G}_0$ at the end. It is negative during $0 < T < T'$, where

$$T' = \frac{3}{2\sqrt{N}}\sinh^{-1}\left[\frac{k^2}{16\pi k'\mathrm{G}_0\sqrt{\frac{24\pi k'\mathrm{G}_0}{N}}}\right].$$

Deceleration parameter $q$ is 1 initially and $q$ tends to $-1$ asymptotically.

For the model (34), we have

$$\rho = \frac{k'}{(\frac{24\pi k' G_0}{-N})\sin^2(\frac{2\sqrt{-N}}{3}T)},$$

$$\theta = \sqrt{-N}\cot\left(\frac{2\sqrt{-N}}{3}T\right),$$

$$\sigma = \frac{k}{\sqrt{3}}\frac{1}{(\sqrt{\frac{24\pi k' G_0}{-N}})^{3/2}\sin^{3/2}(\frac{2\sqrt{-N}}{3}T)},$$

$$G = G_0 - \frac{k^2}{16\pi k'}\frac{1}{\sqrt{\frac{24\pi k' G_0}{-N}}\sin(\frac{2\sqrt{-N}}{3}T)},$$

$$\Lambda = \frac{N}{3} + \frac{k^2/6}{(\sqrt{\frac{24\pi k' G_0}{-N}})^3\sin^3(\frac{2\sqrt{-N}}{3}T)},$$

$$q = -1 + 2\frac{1}{\cos^2(\frac{2\sqrt{-N}}{3}T)}.$$

The model starts with a big-bang at $T = 0$ and expands till $T = 3\pi/(4\sqrt{-N})$ with $\theta = 0$ there. After that it starts contracting and collapses ultimately at $T = 3\pi/(2\sqrt{-N})$. $\rho$ is infinite initially, takes the value $-N/(24\pi G_0)$ at $T = 3\pi/(4\sqrt{-N})$, and becomes infinite again at the end of the model. $\Lambda$ is infinite both initially and finally, and takes a finite value at $T = 3\pi/(4\sqrt{-N})$. G is $-\infty$ both at the start and the end of the model, and has a finite value at $T = 3\pi/(4\sqrt{-N})$. $q$ is 1 both initially and finally, and becomes infinite at $T = 3\pi/(4\sqrt{-N})$. It is interesting to note that $\theta$ behaves periodically with the period $3\pi/(2\sqrt{-N})$, the description (as above) being just during one such period.

## 4 Conclusion

The evolution of an anisotropic universe given by Bianchi type-I cosmological model is studied in the presence of a perfect fluid source with time-dependent gravitational and cosmological 'constants'. The perfect fluid is chosen to obey the barotropic equation of state. We assume the gravitational constant G varies in such a way that G approaches a constant value at late times. The resulting models (28–30), (32) and (33) approach isotropy in the later epochs and cosmological term $\Lambda$ tends asymptotically to a genuine cosmological constant. The models (31) or (34) end in a big crunch or oscillate indefinitely. The behaviour of the universe in these models is determined by the cosmological term $\Lambda$; this term has the same effect as a uniform mass density $\rho_{eff} = -\Lambda/(4\pi G)$, which is constant in space. A positive value of $\Lambda$ corresponds to a negative effective mass density (repulsion) and a negative value of $\Lambda$ corresponds to a positive mass density (attraction). Hence we expect that in the universe with a positive value of $\Lambda$, the expansion will tend to accelerate; whereas in the universe with negative value of $\Lambda$, the expansion will slow down, stop and reverse.

In some of the models, expansion changes from decelerating phase to accelerating one. Recent observations of distant supernovae imply, in defiance of expectation, that the universe growth is accelerating, contrary to what has been assumed that the expansion is slowing down due to gravity. The cosmological constant in these models is decreasing with time approaching a small positive value at the present epoch. Such values are supported by the results from recent supernova Ia observations recently obtained by the High-Z-Supernova Team and Supernova Cosmological Project [15, 16, 24–27, 34]. Our derived models confirm these recent experimental results by showing that the universe now is definitely in the stage of accelerating expansion. These models asymptotically tend to de Sitter universe. For stiff matter, both $\Lambda$ and G turn out to be simply constant for all time.

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